Quantitative Design of Observational Networks for the Arctic

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Sea Ice Workshop - Airborne-based observations of sea ice thickness and snow depth, GSFC, January 2013
Quantitative Network Design

Network Configuration → QND shell → Assimilation System → Model → uncertainty in target quantity
Modelling: NAOSIM

- Sea Ice/Ocean model
- Time step: 1/2 hour
- 0.5 x 0.5 degree hor. res., rotated
- 20 vertical layers
- Model domain: north of about 50°N
- Forcing: daily NCEP reanalysis (but also: JRA25, ERAinterim)

Kauker et al. (2005)
Variational Data Assimilation

Notation:
- $s$: state vector
  (ocean: $u', v', s, t\text{pot}, \Phi$; ice: $h, a, h\text{sn}$)
- $t$: time
- $d$: vector of observations
- $\sigma$: vector observational uncertainties

Principle:
- define vector of control variables $x$, e.g.,
  - forcing/boundary conditions ($f$)
  - Initial state ($s0$)
  - internal model parameters ($p$)
- define quality of fit by cost function:
- minimise $J(x)$ by variation of $x$

$$J(x) = \frac{1}{2}((M(x) - d)^T C_d^{-1} (M(x) - d) + (x - p)^T C_p^{-1} (x - p))$$

uncertainty for obs. term

uncertainty for prior term
Quantitative Network Design

Which Error bar in x is consistent with the error bars in data?
And how does this error bar in $x$ project onto error bars for target quantities of interest?
Uncertainty for target in 2 steps

\[ J(x) = \frac{1}{2} (x - x_{pr})^T C_{pr}^{-1} (x - x_{pr}) + \frac{1}{2} \sum_{i=1,nd} \left( \frac{M_i(x) - d_i}{\sigma_{di}} \right)^2 \]

\[ \frac{d^2 J(x)}{dx^2} = C_{pr}^{-1} + \sum_{i=1,nd} \frac{1}{\sigma_{di}^2} \frac{d^2}{dx^2} (M_i(x) - d_i)^2 \]

- Hessian independent of \( x \) for linear model
- For synthetic data use \( d = M(x) \).
- Decomposes nicely, can precompute model contribution

\[ C_{po} \approx \frac{d^2 J(x_{po})}{dx^2}^{-1} \]

\[ \sigma_y \approx \frac{dy(x_{po})}{dx} C_{po} \frac{dy(x_{po})}{dx}^T \approx \frac{dy(x_{po})}{dx} \frac{d^2 J(x_{po})}{dx^2}^{-1} \frac{dy(x_{po})}{dx}^T \]

For details on methodology see Kaminski and Rayner, 2008
Kaminski et al., 2012a,b
Simple Test

Model Setup:
• coarse 2 degree resolution
• simulation starting on Jan 1, 2007
• simulation period: 1 month
Simple Test

Control Variables:
• initial temperature ocean
• 2-meter atmospheric temperature
• surface wind stress, x direction
• kappa_m (constant in ocean model)
• kappa_h (constant in ocean model)
• pstar (constant in ice model)
• h0 (constant in ice model)
Example

Available Target Quantities:
• Ekin: average ocean Kinetic Energy
• T_mean: mean ocean temperature
• S_mean: mean ocean salinity
• V : integrated ice volume
• A : integrated ice area

Averages computed over January 2007
Simple Test

Available Data Streams:
• a: ice concentration
• h: ice thickness
• hsn: snow thickness

All data streams available
• over each model grid cell
• for each day in January
• with variable data uncertainty
Simple Test

Aircraft Sections:

Beaufort - Kara

East Siberian - Lincoln Sea

Pole - Kara Sea

All sections observe

• On January 15
• Ice thickness of entire 2x2 deg. grid cells with sigma of 30 cm
• Snow thickness of entire 2x2 deg. grid cells with sigma of 30 cm
Uncertainty Reduction

![Bar chart showing uncertainty reduction in percent for different regions and variables (V, T). The chart compares Baufort-Kara, East Sib.-Lincoln, and Pole-Kara regions.](chart.png)
Discussion

• Simple test provides only rough assessment in coarse model with short control vector

• Technique is powerful, can also do
  – remotely sensed observations (see ACCESS newsletter #4)
  – hydrographic observations
  – can help to assess the complementary of observational data streams (Kaminski et al., 2012a)
  – can be used to assess the benefit of a planned space mission (Mission Benefit Analysis, see Kaminski et al., 2012b)
  – can assist the design of an in situ observing strategy